TOPOLOGY AND ARITHMETIC OF MODULI SPACES OF CURVES

1. MINICOURSES

FRANCIS BROWN

RICHARD HAIN
TOPOLOGY, HODGE THEORY AND MOTIVES

This mini course comprises three relatively independent lectures. The main theme will be how the topology of a variety affects and is affected by motives associated with it.

LECTURE I. THE GOLDMAN–TURAEV LIE BIALGEBRA; IS IT MOTIVIC?

The Goldman–Turaev Lie bialgebra of an oriented 2-manifold X is a Lie bialgebra structure on the free abelian group spanned by the conjugacy classes of its fundamental group. Its structure encodes how isotopy classes of immersed loops on X intersect each other and themselves. When X is a smooth complex curve, a suitable completion of the GT-Lie bialgebra carries a natural mixed Hodge structure. The bracket and cobracket are both morphisms (after a suitable twist). This raises the question of whether (when X is defined over a number field) of whether the bracket and cobracket are motivic and, if so, how they are related to algebraic cycles.

I will define the Goldman bracket and Turaev cobracket, as well as their extensions by Kawazumi and Kuno. This is elementary and beautiful surface topology. I'll survey what is known about the MHS and Galois actions on the completed GT-Lie bialgebra and its connections to the Johnson homomorphism.

LECTURE II. THE CERESA CYCLE, ITS NORMAL FUNCTION AND ITS RANK

Normal functions are certain holomorphic sections of families of "intermediate jacobians". They were first defined by Griffiths over 40 years ago to study homologically trivial algebraic cycles and were motivated by work of Poincaré. I will define normal functions and explain how they arise from families of homologically trivial cycles.

The genus $g > 2$ Ceresa cycle is a family of homologically trivial cycles in the universal jacobian over the moduli space of genus g curves. It gives rise to a normal function which is an algebro-geometric (even motivic) incarnation of the Johnson homomorphism. I will define the rank of a normal function, a measure of its non-triviality, and sketch a proof that the generic rank of the normal function of the genus g Ceresa cycle is $3g-3$, the maximum possible. This implies that the maximal power of the curvature form of the associated biextension line bundle is a volume form on the moduli space of curves of compact type and a positive measure on its Deligne–Mumford compactification. One motivation for this work comes from work of Ziyang Gao and Shou-Wu Zhang on the Arakelov theory of moduli spaces of curves.

LECTURE III. HECKE ACTIONS ON LOOPS AND PERIODS OF ITERATED SHIMURA INTEGRALS

Iterated Shimura integrals are iterated integrals of classical modular forms. They are elements of the coordinate ring of the relative unipotent completion of $\mathrm{SL}_2(\mathbb{Z})$, which we regard as the fundamental group of the modular curve. Francis Brown has proposed that the coordinate ring of the appropriate relative completions of $\mathrm{SL}_2(\mathbb{Z})$ generate the (conjectural) tannakian category of mixed modular motives — the category of mixed motives generated by the motives of classical modular forms.

The goal of this talk is to explain how the classical Hecke operators act on the free abelian group generated by the conjugacy classes of $\mathrm{SL}_2(\mathbb{Z})$ and, dually, on those elements of the coordinate ring of the relative completion of $\mathrm{SL}_2(\mathbb{Z})$ that are constant on conjugacy classes. The construction uses only elementary topology.

This Hecke action commutes with the natural Galois action and each Hecke operator is a morphism of mixed Hodge structure. One surprising fact is that, while the Hecke operators T_N and T_M (acting on conjugacy classes) commute when N and M are relatively prime, T_p and T_{p^2} do not commute for any prime p. Consequently, the corresponding Hecke algebra is not commutative, in contrast with the classical case.

2. TALKS

ANTON ALEKSEEV

OMID AMINI

CAREL FABER

MARGARIDA MELO

FLORIAN NAEF

NOEMA NICOLUSSI

DAN PETERSEN
A SIMPLE PROOF OF THE MUMFORD CONJECTURE

Andrea Bianchi recently gave a new proof of Mumford's conjecture on the stable rational cohomology of the moduli space of curves (first proven by Madsen and Weiss). I will explain a streamlined and simplified version of Bianchi's argument. (Joint with Ronno Das)

ULRIKE TILLMANN

HOMOLOGY STABILITY FOR GENERALISED HURWITZ SPACES AND ASYMPTOTIC MONOPOLES

Configuration spaces have played an important role in mathematics and its applications. In particular, the question of how their topology changes as the cardinality of the underlying configuration changes has been studied for some fifty years and has attracted renewed attention in the last decade.

While classically additional information is associated "locally" to the points of the configuration, there are interesting examples when this additional information is "non-local". With Martin Palmer we have studied homology stability in some of these cases, including Hurwitz space and moduli spaces of asymptotic monopoles.

ORSOLA TOMMASI

KAREN VOGTMANN

THOMAS WILLWACHER

CYCLIC MODEL FOR THE DG DUAL OF THE BV OPERAD

I describe a small cyclic homotopy operad model for the dg (Koszul-)dual of the BV operad. As an application, one can show that the Feynman transform of BV and of its dg dual are quasi-isomorphic.

The talk is based on `https://arxiv.org/abs/2311.09037`.